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The yield function of a ⟨100⟩ textured polycrystalline CuCoSi alloy wire for simultaneous torsion and extension

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⟨1 00⟩ textured Cu–0.64 at % Co–0.48 at % Si alloy wires were studied by simultaneous torsion and extension as well as by metallographic and X-ray investigations. The yield function due to the ⟨1 0 0⟩ texture was computed for the case of simultaneous torsion and extension. The computed yield function is in good agreement with the experimental results.

1. Introduction

It is generally assumed, that in a polycrystalline metal the grains are randomly oriented, consequently the von Mises or the Tresca yield conditions can be applied for the description of plastic deformation, e.g. in a polycrystalline wire sample. In many cases, however, a recrystallization texture develops during a high-temperature heat treatment, e.g. a solution treatment. The recrystallization texture in copper wires can be a mixture of ⟨1 0 0⟩ and ⟨1 1 1⟩ type textures in the direction of the wire axis or it can have pure ⟨1 0 0⟩ character [1].

The texture in polycrystalline copper or in copper-base alloys can be investigated by the statistical analysis of the orientation of twin boundaries which can easily be observed in metallographic pictures. The aim of this paper is the experimental and theoretical determination of the yield function for simultaneous torsion and extension in the case of a ⟨1 0 0⟩ textured Cu–0.64 at % Co–0.48 at % Si wire sample.

2. Experimental details

The samples investigated were 120 mm long wires of 1.5 mm diameter. The specimens were solution treated by annealing for 1 h in vacuum better than 10^{-2} Pa at 1000° C which produced a coarse-grained polycrystalline structure with an average grain size of about 100 μm. After annealing the samples were quenched in a silicon oil bath. Electrical resistivity measurements performed at 78 K confirmed that after this heat treatment the alloying atoms were completely dissolved in the copper matrix. The grain structure of the wire was investigated by metallography and X-ray back-reflection on sections parallel and perpendicular to the wire axis. On the metallographic pictures numerous twin boundary lines have been observed. The statistical investigation of the direction of these twin boundary lines to the wire axis led to the conclusion that the texture is of pure ⟨1 0 0⟩ character, which was confirmed by X-ray results as well.

The simultaneous torsion and extension was investigated by the method described in [2]. The following parameters were measured simultaneously: tensile stress (σ), torque (M), angles of elastic and plastic torsion as well as plastic elongation. Plastic flow was reached by continuously increasing the torque while keeping the tensile stress at a constant value. The torsion-extension measurements were carried out at room temperature.

3. Characterization of the texture on the basis of metallographic and X-ray investigations

According to the metallographic pictures of sections parallel to the axis the great majority of

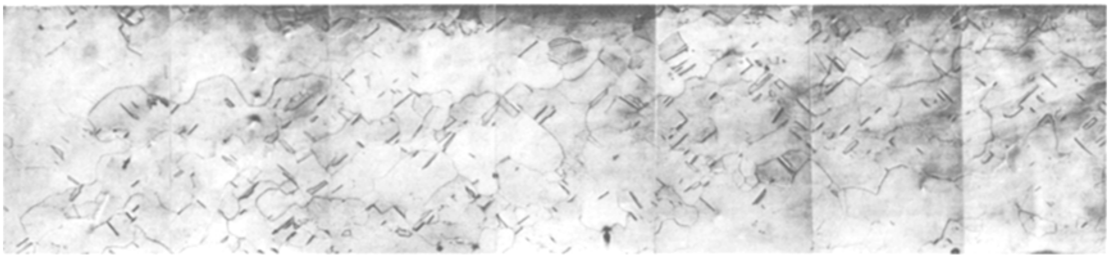

Figure 1 Metallographic picture from the longitudinal section of the specimen.

the twin boundaries include a near 45° angle with the axis of the wire (Fig. 1). Since the twin boundaries in a face centred cubic (f c c) metals are situated along the {1 1 1} planes the straight lines observed on the metallographic pictures indicate the lines of intersection of the {1 1 1} planes with the section plane investigated. Consequently the twin boundaries in the metallographic pictures can be used as indicators of grain orientation and texture. For the 276 twin boundaries which can be seen in Fig. 1, the angle η included by the individual twin boundary lines and by the longitudinal axis of the wire was determined. The obtained frequency distribution of the angle η is shown in Fig. 2. The distribution shows a sharp maximum at 35°, smaller angles, however, were observed only in a very few cases.

Let us consider now a textured material in which one of the ⟨1 0 0⟩ directions in any grain is parallel to the axis of the wire but otherwise the grains may occupy any position around this direction. Assuming a totally random distribution for this latter position the frequency distribution of the angle of the plane parallel to the wire axis and of the {1 1 1} planes has been simulated by a computer. The result is shown in Fig. 3. Comparing this computed distribution function with the experimentally observed distribution in Fig. 2 it can be established that the latter one is really characteristic of a ⟨1 0 0⟩ type texture.

In a cross-section perpendicular to the wire axis the twin boundaries are randomly positioned (Fig. 4). It can be observed that in this section if twin boundaries corresponding to different ⟨1 1 1⟩ directions in a given grain are found, then these are perpendicular to each other, which is clearly a consequence of the ⟨1 0 0⟩ texture.

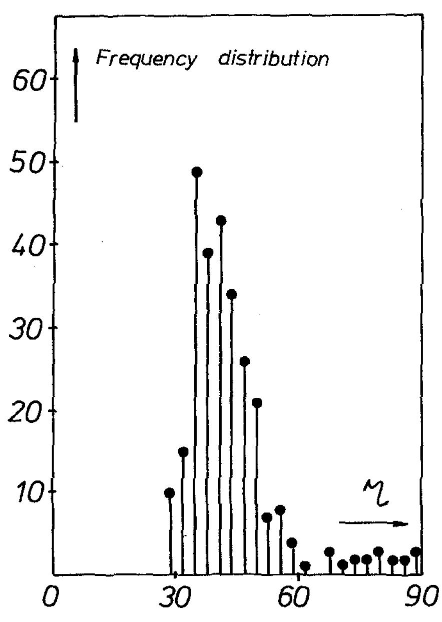

Figure 2 Frequency distribution of the angle, η, included by the twin boundary lines in Fig. 1 and the longitudinal axis of the specimen.

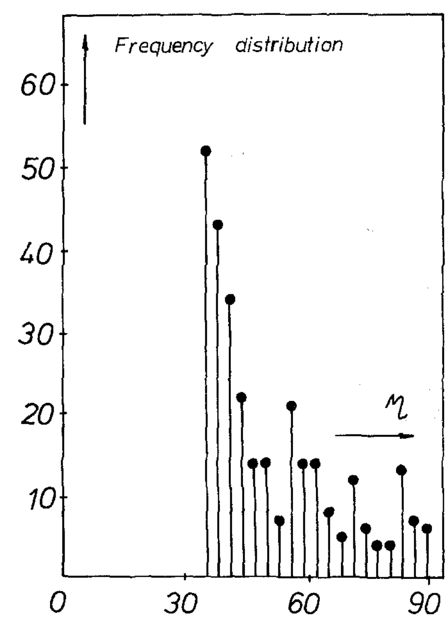

Figure 3 Computer simulation of angle distribution for ⟨1 0 0⟩ texture.

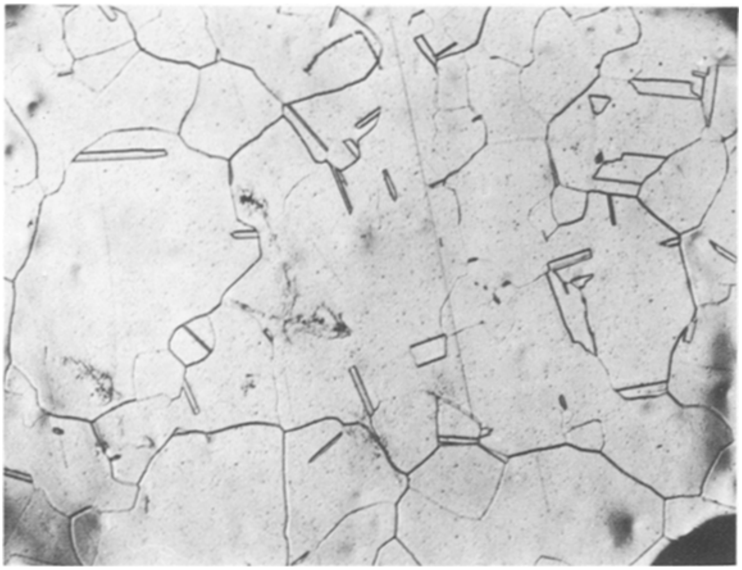

Figure 4 Metallographic picture from a cross-section perpendicular to the wire axis.

The conclusions drawn from the metallographic observations have been confirmed by X-ray investigations. The X-ray back-reflection method used is shown schematically in Fig. 5. Corresponding to the ⟨100⟩ texture the reflection dots were aligned along hyperbolic lines which were identified as reflections from the crystallographic planes indicated in the figure.

4. Determination of the yield function

Plastic deformation is initiated in a given direction of a slip plane in a crystal if the resolved shear stress, τ_{ik}, attains the critical value, τ_r, i.e. if

$$\tau_{ik} = |(\hat{\sigma}\mathbf{n}_i)\mathbf{g}_k| = \tau_r \quad (1)$$

where $\mathbf{n}_i$ is the normal vector of the slip plane, $\mathbf{g}_k$ is the unit vector in the slip direction and $\hat{\sigma}$ is the stress tensor. The shear stresses operating in the 12 slip systems of an f c c lattice at given external forces are generally strongly different. The plastic flow of polycrystals needs at least five different slip systems to be activated in each grain. Namely the deformation tensor, because of its symmetry, contains at most six different elements among which the $\epsilon_{11} + \epsilon_{22} + \epsilon_{33} = 0$ relation (constancy of volume) holds. As a result of this any plastic deformation can be described by a maximum of five independent data. Consequently any plastic deformation can be obtained by the simultaneous operation of five slip systems.

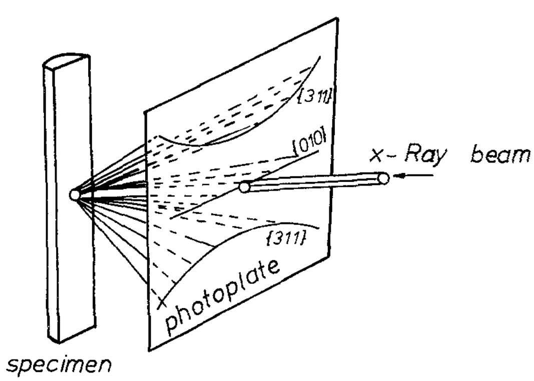

Figure 5 The scheme of the X-ray back-reflection measurements.

We shall identify the external stress necessary to initiate plastic flow in a given grain with the one when the average of the five largest stresses of the τ_{ik}'s operating in all the possible slip systems attains the critical stress.

To determine the yield condition for the ⟨100⟩ texture described in section 3 we have to take into account that one of the ⟨100⟩ directions is parallel to the z-axis in each grain. The Thompson tetrahedron for this situation is shown in Fig. 6. It can be seen that another characteristic of the texture in question is that for a grain with a given coordinate, ϕ, the β angle may have any value between 0 and 360°.

In a rectangular coordinate system the stress tensor for simultaneous torsion and extension is the following:

$$\hat{\sigma} = \begin{pmatrix} 0 & 0 & -\tau\sin\phi \\ 0 & 0 & \tau\cos\phi \\ -\tau\sin\phi & \tau\cos\phi & \sigma \end{pmatrix} . \quad (2)$$

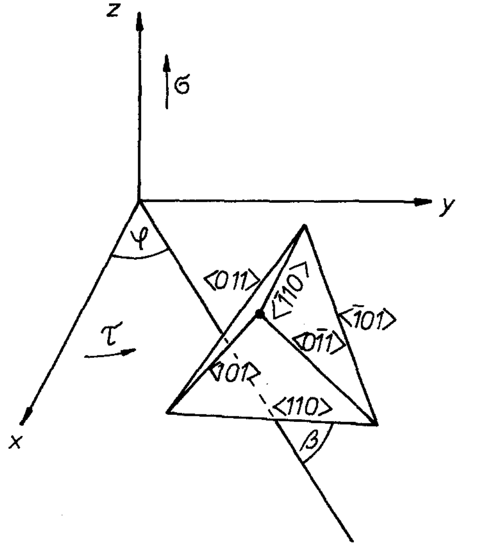

Figure 6 The position of the Thompson tetrahedron in the case of ⟨1 0 0⟩ texture.

Using Equation 1 and the geometrical symmetry two types of expressions for the shear stresses operating in the different slip systems can be obtained:

$$\tau(1) = \frac{\tau}{2(3)^{1/2}}\left|(\pm)\cos\beta\,(\pm)\sin\beta + 2^{1/2}\,\frac{\sigma}{\tau}\right| \quad (3)$$

$$\tau(2) = \begin{cases} \dfrac{\tau}{3^{1/2}}\,|\cos\beta| \\ \\ \dfrac{\tau}{3^{1/2}}\,|\sin\beta| \end{cases} \quad (4)$$

According to Fig. 6 the $\tau(1)$ type stresses are operating along the slip vectors [0 1 1], [$\bar{1}$0 1], [0$\bar{1}$1], [1 0 1]. In these slip systems both the tensile and the torsional stresses are active. The $\tau(2)$ type stresses are operating along the slip vectors [1 1 0] and [$\bar{1}$1 0], whose slip systems are activated only by the torsional stress.

For the computation of the yield stresses in the case of a given value of σ/τ a value of the angle β was selected randomly and the τ_{ik}/τ quantity was calculated for each of the 12 slip systems. The critical shear stress of the grain is calculated from the average of the five largest τ_{ik}/τ quantity. Performing this computation for several hundred grains the mean value of these stresses can be considered as the shear stress of the polycrystalline material. It is worth mentioning that the highest and the lowest values obtained for the different grains did not deviate from each other by more than 8%. Even if the computation is carried out for only hundred grains the standard deviation of the mean value does not exceed 0.05%.

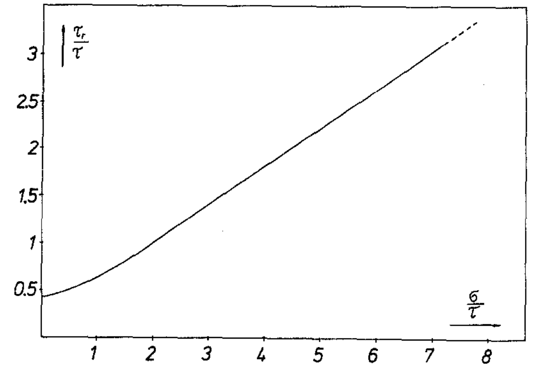

Figure 7 The ratio of the critical shear stress and of the torsional stress against the ratio of the tensile and of the torsional stress.

The ratio of the critical shear stress and of the torsional stress as a function of the ratio of the tensile and torsional stresses, σ/τ is shown in Fig. 7. For simple torsion $\sigma/\tau = 0$, while for simple tension $\sigma/\tau = \infty$. For $\sigma/\tau > 2$ this relationship is linear. From the slope of the linear part of the curve in Fig. 7 the macroscopic yield stress for the case of pure tension can be given as

$$\sigma_f = \frac{1}{0.408}\tau_r = 2.45\tau_r \quad (5)$$

and from the ordinate at $\sigma/\tau = 0$ of the curve the yield stress for pure torsion can be expressed as

$$\tau_f = \frac{1}{0.430}\tau_r = 2.33\tau_r. \quad (6)$$

The Taylor factor in Equation 5 is characteristic also to the extension of the ⟨1 0 0⟩ oriented f c c single crystals, for which the same result was obtained previously [3].

To determine the yield condition for simultaneous torsion and extension the relationship between σ and τ at the initiation of plastic flow has to be established. Denoting the τ_r/τ values shown in Fig. 7 by $f(\sigma/\tau)$ and with the help of Equations 5 and 6 we can write:

$$\frac{\sigma}{\sigma_f} = 0.408\,\frac{\sigma}{\tau_r} = \frac{0.408\,\sigma}{f(\sigma/\tau)\,\tau} \quad (7)$$

and

$$\frac{\tau}{\tau_f} = 0.430\,\frac{\tau}{\tau_r} = \frac{0.430}{f(\sigma/\tau)}. \quad (8)$$

It can be seen that at the yielding both σ/σ_f and τ/τ_f are functions of σ/τ. Using the curve in Fig. 7

both σ/σ_f and τ/τ_f can be calculated from these equations. The relationship between σ/σ_f and τ/τ_f is the yield function which is shown by the continuous line in Fig. 8. For $\tau/\tau_f < 1/2$ the yield function is linear, since in this region the torsional stress is not large enough to activate the second type of slip system (Equation 4). So if $\sigma/\tau > 2$ only the slip systems of the first type (Equation 3) become activated. The linearity of the yield function in this region is a consequence of the fact that in this region the critical shear stress is a linear combination of the torsional and the tensile stresses:

$$\tau_r = \overline{\tau(1)} = \frac{1}{2(3)^{1/2}} \overline{|(\pm)\cos\beta(\pm)\sin\beta\ \tau + (2)^{1/2}\sigma|} \qquad (9)$$

and the trigonometric average is independent of the stresses.

5. Experimental results

The experimental points of the yield function (open circles in Fig. 8) were obtained by the following method. First the specimen was deformed in pure tension to obtain at least 0.2% plastic elongation after unloading. The tensile stress necessary for this is σ_f (point A in Fig. 9). After this with zero tensile stress a very small plastic torsion was applied so that the work hardening in this process is negligible (point B). From the experimentally measured value of the torque, M, the yield strength for pure shearing was determined

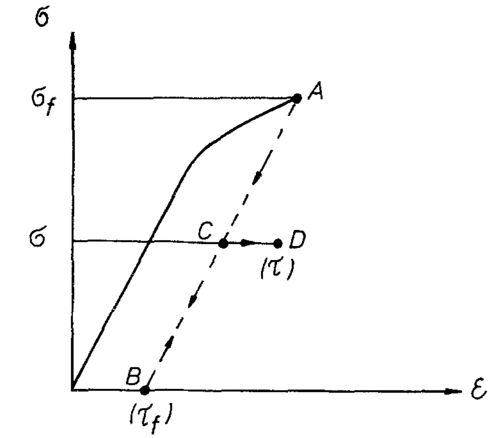

Figure 9 The scheme for the experimental determination of the yield function.

by the $\tau_f = 2M/\pi a^3$ relation [4] where a is the radius of the specimen. Following this a $\sigma < \sigma_f$ tensile stress was applied (point C) and by increasing the torque the threshold of plastic deformation was reached at point D where simultaneous plastic torsion and extension was initiated. The torsional stress necessary for yielding is τ. The corresponding experimental points of σ/σ_f and τ/τ_f are shown in Fig. 8.

According to Equations 5 and 6 for the ⟨100⟩ texture investigated the yield strength for pure shear and that for pure extension is nearly the same. To investigate the validity of this result the yield strength of a specimen was increased by deforming it in pure tension. After interrupting the tensile deformation and unloading the specimen the yield strength of pure torsion was measured. The experimentally obtained σ_f/τ_f points against tensile elongation, ϵ, are shown in Fig. 10. It can be seen that the experimental points are near to the $\sigma_f/\tau_f = 1.054$ value obtained from Equations 5 and 6.

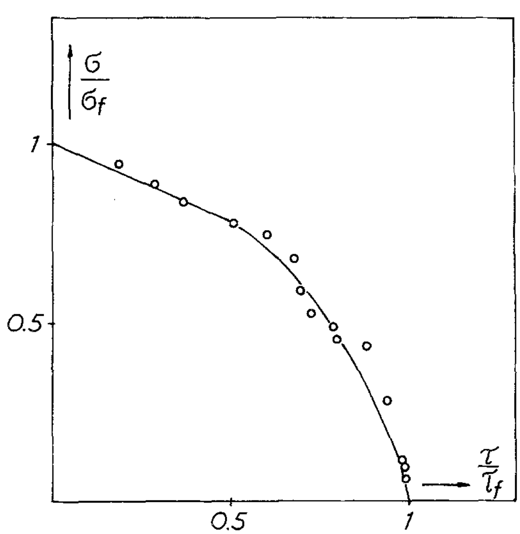

Figure 8 The yield function of the ⟨1 0 0⟩ texture (continuous line).

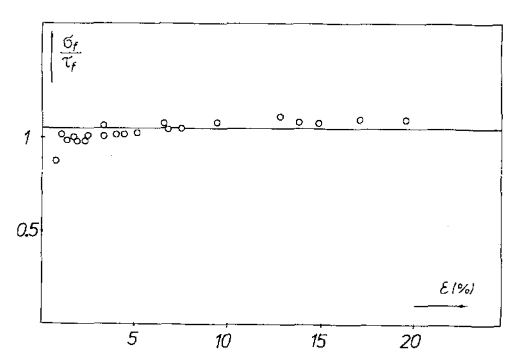

Figure 10 The ratio of the yield strength of the pure tension and that of pure torsion against tensile elongation.

6. Conclusion

The yield function of a ⟨100⟩ textured polycrystalline CuCoSi alloy wire was calculated by applying computer simulation to determine the individual shear stresses of the different grains. The results obtained are in good agreement with the experimental observations. The method applied can be extended to more complicated distributions of grain orientations.

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